$\boxed{1.a}$ (0.2)

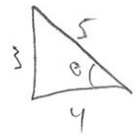
$$1+j = (r \angle \theta)$$

$$r = \sqrt{1^2+1^2} = \boxed{\sqrt{2}} = \boxed{1.414}$$

$$\theta = \frac{\pi}{4} \approx \boxed{0.7854}$$

$\boxed{1.b}$

$$-4+j3 = (r \angle \theta)$$



$$r = \boxed{5}$$

$$\theta = \boxed{\frac{\pi}{2} - \tan^{-1}\left(\frac{3}{4}\right) \approx 2.498}$$

$\boxed{1.c}$

$$(1+j)(-4+j3) = \left(\sqrt{2} \angle \frac{\pi}{4}\right)(5 \angle 2.498) = \left(5\sqrt{2} \angle \frac{\pi}{4} + 2.498\right) =$$

$$= (5\sqrt{2} \angle 3.283 - 2\pi) = \boxed{(7.071 \angle -3.000)}$$

$\boxed{1.d}$

$$e^{j\pi/4} + 2e^{-j\pi/4} = \cos\frac{\pi}{4} + j\sin\frac{\pi}{4} + 2\cos\frac{\pi}{4} - j2\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} + \frac{2}{\sqrt{2}} - \frac{j2}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(3-j) = (r \angle \theta)$$

$$r = \frac{1}{\sqrt{2}}\sqrt{3^2+1} = \frac{\sqrt{10}}{\sqrt{2}} = \boxed{\sqrt{5}} \approx \boxed{2.236}$$

$$\theta = \tan^{-1}\left(-\frac{1}{3}\right) \approx \boxed{-0.3218}$$

$\boxed{1.e}$

$$e^{j}+1 = 1+\cos(1) + j\sin(1) = (r \angle \theta)$$

$$r = \sqrt{(1+\cos 1)^2 + \sin^2 1} = \sqrt{\cos^2 1 + \sin^2 1 + 2\cos^2 1 + 1} = \boxed{\sqrt{2(\cos^2 1 + 1)} \approx 1.755}$$

$$\theta = \tan^{-1}\left(\frac{\sin(1)}{\cos(1)+1}\right) = \boxed{0.5000}$$

$\boxed{1.f}$

$$\frac{1+j}{-4+3j} = \frac{\left(\sqrt{2} \angle \frac{\pi}{4}\right)}{(5 \angle 2.214)} = \boxed{\left(\frac{\sqrt{2}}{5} \angle \frac{\pi}{4} - 2.498\right) = (0.2828 \angle -1.713)}$$

2.a (B.3)

$$3e^{j\pi/4} = 3\cos\left(\frac{\pi}{4}\right) + 3j\sin\left(\frac{\pi}{4}\right) = \boxed{\frac{3}{\sqrt{2}} + \frac{3}{\sqrt{2}}j} \approx 2.121 + j\,2.121$$

2.b

$$\frac{1}{e^{j}} = e^{-j} = \cos(-1) + j\sin(-1) = \boxed{\cos(1) - j\sin(1)} \approx 0.5403 - j0.8415$$

2.c

$$(1+j)(-4+j3) = (-4-3) + j(-4+3) = \boxed{-7-j}$$

2.d

$$e^{j\pi/4} + 2e^{-j\pi/4} = \cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right) + 2\cos\left(\frac{\pi}{4}\right) - 2j\sin\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}} + \frac{-2}{\sqrt{2}}j = \boxed{\frac{3}{\sqrt{2}} - \frac{1}{\sqrt{2}}j} \approx 2.121 - j0.7071$$

2.e

$$e^{j}+1 = \cos(1) + j\sin(1) + 1 = \boxed{(1+\cos(1)) + j\sin(1)} \approx 1.540 + j0.8415$$

2.f

$$1/2^{j} = 2^{-j} = \exp(-j(\ln(2) + 2\pi jn)) = \exp(-2\pi n - j\ln(2))$$

$$= \boxed{e^{-2\pi n}\cos(\ln(2)) - je^{-2\pi n}\sin(\ln 2)} \qquad n = 0, \pm1, \pm2, \ldots \quad (n \in \mathbb{Z})$$

"principal value" is $n=0$: $\cos(\ln 2) - j\sin(\ln(2)) \approx 0.7692 - 0.6392j$

Assume conjugation relations from #5

3.a) (B.6)

$$Re(jw_1) = \frac{jw_1 + (jw_1)^*}{2} = \frac{jw_1 - jw_1^*}{2}\,\frac{j}{j} = \frac{-(w_1 - w_1^*)}{2j} = -Im(w_1)$$

$\boxed{\text{true}}$

3.b)

$$Im(jw_1) = \frac{jw_1 - (jw_1)^*}{2j} = \frac{jw_1 + jw_1^*}{2j} = \frac{w_1 + w_1^*}{2} = Re(w_1)$$

$\boxed{\text{true}}$

3.c)

$$Re(w_1) + Re(w_2) = \frac{W_1 + W_1^*}{2} + \frac{w_2 + w_2^*}{2} = \frac{1}{2}\left((w_1+w_2) + (w_1^* + w_2^*)\right)$$

$$= \frac{1}{2}\left((w_1+w_2) + (w_1+w_2)^*\right) = Re(w_1+w_2)$$

$\boxed{\text{true}}$

3.d)

$$Im(w_1+w_2) = \frac{1}{2j}(w_1 - w_1^*) + \frac{1}{2j}(w_2 - w_2^*) = \frac{1}{2j}\left((w_1+w_2) - (w_1+w_2)^*\right)$$

$$= Im(w_1+w_2)$$

$\boxed{\text{true}}$

3.e)

$$Re(w_1 w_2) = \frac{w_1 w_2 + (w_1 w_2)^*}{2}$$

$$Re(w_1)Re(w_2) = \left(\frac{w_1 + w_1^*}{2}\right)\left(\frac{w_2 + w_2^*}{2}\right) = \frac{w_1 w_2 + w_1^* w_2^*}{2} + \frac{w_1 w_2^* + w_1^* w_2}{2}$$

$$= Re(w_1 w_2) + \frac{w_1 w_2^* + (w_1 w_2^*)^*}{2} = Re(w_1 w_2) + Re(w_1 w_2^*)$$

$\boxed{\text{false}}$

3.f)

Consider w_2 such that $Im(w_2) = 0$. Then $\frac{Im(w_1)}{Im(w_2)}$ is undefined.

But $Im\left(\frac{w_1}{w_2}\right) = \frac{1}{2j}\left(\frac{w_1}{w_2} - \left(\frac{w_1}{w_2}\right)^*\right)$

$$= \frac{1}{2j}\left(\frac{w_1}{w_2} - \frac{w_1^*}{w_2}\right) \quad (w_2 \text{ is real})$$

$$= \frac{1}{w_2}\,\frac{1}{2j}(w_1 - w_1^*) = \frac{1}{w_2} Im(w_1)$$

$\boxed{\text{false}}$

(in general: $Im\left(\frac{w_1}{w_2}\right) = \frac{1}{|w_2|^2} Im(w_1 w_2^*)$

4.a (B.11)

$$\cosh(w) = \frac{\exp(w) + \exp(-w)}{2} = \frac{1}{2}\left(\exp(x)\exp(jy) + \exp(-x)\exp(-jy)\right)$$

$$= \frac{1}{2}\left(\exp(x)\cos(y) + j\exp(x)\sin(y) + \exp(-x)\cos(y) - j\exp(-x)\sin(y)\right)$$

$$= \frac{1}{2}\left(\cos(y)(e^{x} + e^{-x}) + j\sin(y)(e^{x} - e^{-x})\right)$$

$$= \cosh(x)\cos(y) + j\sinh(x)\sin(y)$$

4.b

$$\sinh(w) = \frac{1}{2}\left(\exp(+x)\exp(+jy) - \exp(-x)\exp(-jy)\right)$$

$$= \frac{1}{2}\left(e^{x}\cos y + je^{x}\sin y - e^{-x}\cos(y) + je^{-x}\sin(y)\right)$$

$$= \frac{1}{2}\left(\cos(y)(e^{x} - e^{-x}) + j\sin(y)(e^{x} + e^{-x})\right)$$

$$= \sinh(x)\cos(y) + j\cosh(x)\sin(y)$$

5.a

$$(a^* + b^*) = \mathrm{Re}(a) + -j\,\mathrm{Im}(a) + \mathrm{Re}(b) - j\,\mathrm{Im}(b)$$

$$= (\mathrm{Re}(a) + \mathrm{Re}(b)) - j(\mathrm{Im}(a) + \mathrm{Im}(b))$$

$$= (a+b)^*$$

5.b

$a = a_r + a_i j \qquad b = b_r + b_i j$

$$a^* b^* = (a_r - ja_i)(b_r - jb_i) = (a_r b_r - a_i b_i) - j(a_i b_r + a_r b_i)$$

$$= \left((a_r b_r - a_i b_i) + j(a_i b_r + a_r b_i)\right)^*$$

$$= (ab)^*$$

5.c

$$\exp(a^*) = \sum_{n=0}^{\infty} \frac{(a^*)^n}{n!} = \underbrace{\sum_{n=0}^{\infty} \frac{(a^n)^*}{n!}}_{\text{(part b)}} = \underbrace{\left(\sum_{n=0}^{\infty} \frac{a^n}{n!}\right)^*}_{\text{(part a)}} = \exp(a)^*$$

Also: $\exp(a^*) = \exp(a_r - ja_i) = e^{a_r}(\cos(-a_i) + j\sin(-a_i)) = e^{a_r}(\cos(a_i) - j\sin(a_i))$

$$= e^{a_r}(\cos(a_i) + j\sin(a_i))^* = \exp(a)^*$$

6.a

$$j^j = \exp(j\ln(j)) = \exp(j(\underbrace{\ln|j|}_{\text{modulus}} + j\underbrace{\arg(j)}_{\text{angle(s)}}))$$

$$= \exp\left(j\left(\cancel{\ln(1)}^{0} + j\frac{\pi}{2} + 2\pi j n\right)\right)$$

$$= \exp\left(-\frac{\pi}{2}\right)\exp(-2\pi n), \quad n \in \mathbb{Z}$$

Note: They are all real

$$= e^{-\pi/2} = 0.2079, \quad n = 0$$

6.b

$$1^j = \exp(j\ln(1)) = \exp(j(\cancel{\ln(1)}^{0} + 2\pi j n))$$

Again, all real

$$= \exp(-2\pi n), \quad n \in \mathbb{Z}$$

$$= 1 \quad (n = 0)$$

6.c

$j^0 = 1$ by definition

6.d

$$\sqrt{j} = \exp\left(\frac{1}{2}\ln(j)\right) = \exp\left(\frac{1}{2}\left(j\frac{\pi}{2} + 2\pi j n\right)\right)$$

$$= e^{j\pi/4} e^{\pi j n} = e^{j\pi/4}(-1)^n = \pm e^{j\pi/4}$$

All the different values collapse to two distinct values. They are negatives, just like real square roots.

7]

$$|a+b|^2 = (a+b)(a+b)^* = aa^* + bb^* + ab^* + a^*b$$
$$= |a|^2 + |b|^2 + ab^* + (ab^*)^* = |a|^2 + |b|^2 + 2\,\mathrm{Re}(ab^*)$$

$$j\,|a+jb|^2 = j(|a|^2 + |jb|^2 + 2\,\mathrm{Re}(a(jb)^*))$$
$$= j(|a|^2 + |b|^2 + 2\,\mathrm{Re}(-jab^*)) = j|a|^2 + j|b|^2 + j2\,\mathrm{Im}(ab^*)$$

$$j^2|a+j^2b|^2 = -|a-b|^2 =$$
$$= -(|a|^2 + |b|^2 - 2\,\mathrm{Re}(ab^*)) = -|a|^2 - |b|^2 + 2\,\mathrm{Re}(ab^*)$$

$$j^3|a+j^3b|^2 = -j|a-jb|^2$$
$$= -j(|a|^2 + |b|^2 + 2\,\mathrm{Re}(a(-jb)^*))$$
$$= -j(|a|^2 + |b|^2 + 2\,\mathrm{Re}(jab^*)) = -j|a|^2 - j|b|^2 + j2\,\mathrm{Im}(ab^*)$$

If we sum these, all of the modulus squared terms cancel, while the real/imaginary part terms add.

$$|a+b|^2 + j|a+jb|^2 + j^2|a+j^2b|^2 + j^3|a+j^3b|^2$$

$$= 4\,\mathrm{Re}(ab^*) + 4j\,\mathrm{Im}(ab^*) = 4(ab^*)$$